

Paper Reference 9FM0/4D

Pearson Edexcel

Level 3 GCE

Further Mathematics

Advanced

PAPER 4D: Decision Mathematics 2

Monday 23 June 2025 – Afternoon

Time: 1 hour 30 minutes

Question Paper

**THIS DOCUMENT MUST NOT BE RETURNED TO
PEARSON AT THE END OF THE EXAMINATION.**

YOU MUST HAVE

**Mathematical Formulae and Statistical Tables (Green),
calculator**

YOU WILL BE GIVEN

Diagram Booklet

Answer Booklet

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.

Do not return the Question Paper with the Answer Booklet.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Booklet or in the separate Diagram Booklet – there may be more space than you need.

Do NOT write on this Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

INFORMATION

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

There are **7** questions in this Question Paper.

The total mark for this paper is **75**

The marks for **EACH** question are shown in brackets – use this as a guide as to how much time to spend on each question.

There are two copies of all diagrams.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

1. Refer to Diagram 1 in the Diagram Booklet.

Ravi can choose one of three options, A, B or C, when playing a game.

The profit, in pounds, associated with each outcome, and the corresponding probabilities, are shown in the decision tree in the Diagram Booklet.

Calculate the optimal **EMV to determine Ravi's best course of action.**

You must make your working clear.

(Total for Question 1 is 3 marks)

2. Refer to Diagram 2 in the Diagram Booklet.

Five workers, A, B, C, D and E, are available to complete four tasks, P, Q, R and S

Each worker can be assigned to at most one task, and each task must be done by at most one worker.

Worker B cannot be assigned to task R

The time, in minutes, that each worker takes to complete each task is shown in the table in the Diagram Booklet.

The Hungarian algorithm is to be used to find an allocation that minimises the total time to complete all four tasks.

- (a) Explain how the table should be modified so that the Hungarian algorithm can be applied.**
(2 marks)

(continued on the next page)

2. continued.

(b) Use the Hungarian algorithm to obtain an allocation that minimises the total time.

(6 marks)

(c) Calculate the least total time to complete all four tasks.

(1 mark)

(Total for Question 2 is 9 marks)

3. Refer to Diagram 3 in the Diagram Booklet.

It is a table showing the cost, in pounds, of transporting one unit of stock from each of four supply points, A, B, C and D, to three demand points, P, Q and R

It also shows the stock held at each supply point and the number of units required at each demand point.

A minimum cost solution is required.

- (a) Use the north–west corner method to obtain an initial solution to this transportation problem.**
(1 mark)

(continued on the next page)

3. continued.

(b) Perform one iteration of the stepping–stone method to obtain an improved solution.

You must make your method clear by showing the route and stating the

- shadow costs**
- improvement indices**
- entering cell and exiting cell**

(4 marks)

(c) Formulate the transportation problem as a linear programming problem.

You must define your decision variables and make the objective function and constraints clear.

(5 marks)

(Total for Question 3 is 10 marks)

4. Refer to Diagram 4, Diagram 5 and Diagram 6 in the Diagram Booklet.

Diagram 4 shows a capacitated, directed network. The network represents a system of pipes through which fluid can flow.

The weights on the arcs show the lower and upper capacities for the corresponding pipes, in litres per second.

(a) Calculate the capacity of

(i) cut C_1

(ii) cut C_2

(2 marks)

(b) Using only the capacities of cuts C_1 and C_2 , state what can be deduced about the maximum flow through the system.

(1 mark)

(continued on the next page)

Turn over

4. continued.

A scientist monitors the system of pipes in which a fluid now flows from the source, S , to the sink, T

The scientist initialises the labelling procedure for this system.

The excess and potential backflows for this are shown on the arrows either side of each arc in Diagram 5 in the Diagram Booklet.

(c) State the value of the initial flow.

(1 mark)

(d) Use the labelling procedure to find a maximum flow through the network.

You must list each flow–augmenting route you use, together with its flow.

(3 marks)

(e) Use your answer to part (d) to find a maximum flow pattern for this system of pipes and draw it on Diagram 6 in the Diagram Booklet.

(1 mark)

(continued on the next page)

Turn over

4. continued.

(f) Prove that the answer to part (e) is optimal.

(3 marks)

(Total for Question 4 is 11 marks)

5. Refer to Diagram 7 and Diagram 8 in the Diagram Booklet.

Anvi makes boats during the winter months.

She can make up to five boats each month.

If she builds more than three boats in any one month, she must hire an assistant at a cost of £400 for that month.

In any month in which boats are made, the overhead costs are £60 for each boat made that month.

A maximum of three boats can be held in storage at the end of each month, at a cost of £70 per boat per month.

Boats must be delivered at the end of each month.

The order schedule for boats is shown in Diagram 7 in the Diagram Booklet.

(continued on the next page)

5. continued.

There are no boats in storage at the beginning of November.

Anvi plans to have no boats left in storage after the end of the March delivery.

(a) Use dynamic programming to determine the production schedule that minimises the costs given on the previous page.

**Complete the working in Diagram 8 in the Diagram Booklet and state the minimum cost.
(12 marks)**

The assistant asks to work in November, and Anvi agrees.

**(b) Given that more than three boats are now made in November, state the minimum additional cost of producing all the required boats to the schedule shown in Diagram 7 in the Diagram Booklet.
(1 mark)**

(Total for Question 5 is 13 marks)

Turn over

6. Refer to Diagram 9 and Diagram 10 in the Diagram Booklet.

A two person zero-sum game is represented by the pay-off matrix for player **A** shown in Diagram 9

- (a) Verify that there is no stable solution to this game.
(2 marks)

Let player **A** play option **X** with probability p

- (b) Use a graphical method to find the optimal value of p and hence find the best strategy for player **A** in this game.
(6 marks)

(continued on the next page)

6. continued.

A third option, **Z**, is added to player **A**'s options. Option **Z** has the pay-offs shown in the matrix in Diagram 10 in the Diagram Booklet.

Player **A** now intends to make a random choice between options **X**, **Y** and **Z**, choosing option **X** with probability **x**, option **Y** with probability **y** and option **Z** with probability **z**

Player **A** decides to use the Simplex algorithm to find the optimal values of **x**, **y** and **z**

(c) Determine an initial Simplex tableau to solve this 3×3 game, making your variables clear. The incomplete Simplex tableau is on page 37 of the Answer Booklet.

(4 marks)

(continued on the next page)

6. continued.

In the optimal Simplex tableau for this 3×3 game,

$$x = 0 \text{ and}$$

$$y = \frac{5}{7}$$

- (d) By considering the values of the two games, determine how much better the 3×3 game (with option **Z**) is for player **A** compared with the 2×3 game (without option **Z**)

You must make your reasoning clear.

(3 marks)

(Total for Question 6 is 15 marks)

7. A sequence $\{u_n\}$, where $n \geq 0$, satisfies the first order recurrence relation (*)

$$u_{n+1} = \alpha u_n - \beta(k^n) \quad (*)$$

where α , β , k are non-zero constants and $\alpha \neq k$

- (a) Determine, in terms of α , β , k and n , the general solution of this recurrence relation.
(4 marks)

(continued on the next page)

7. continued.

Given that

$$u_0 = L \text{ and}$$

$$u_T = 0, \text{ where } T \text{ is a constant,}$$

(b) show that a particular solution of the recurrence relation is

$$u_n = \frac{Lk^n \left(1 - \left(\frac{k}{\alpha} \right)^{T-n} \right)}{1 - \left(\frac{k}{\alpha} \right)^T}$$

(5 marks)

(continued on the next page)

7. continued.

Taylor borrows a sum of money, L , to buy a house.

Taylor arranges a loan which enables them to increase their repayment as their income grows.

The terms of the loan require Taylor's repayments to be made once a year.

On each anniversary of the loan being taken out, 6% interest is charged on the outstanding balance and Taylor makes an annual repayment of the total loan.

Taylor's total annual repayment of the loan will increase by 4% each year and the whole debt will be cleared in exactly 40 years.

The recurrence relation (*) can be used to model this situation.

(continued on the next page)

7. continued.

(c) (i) State the value of α

(ii) State the value of k

(iii) State the relevance of β in (*)

(3 marks)

(d) Show that after 30 years, according to the model, the outstanding debt is larger than the original loan.

(2 marks)

(Total for Question 7 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER